Optimal Nonlinear Pricing, Bundling Commodities and Contingent Services

Marion Podesta, Jean-Christophe Poudou

Optimal Nonlinear Princing, Bundling Commodities and Contingent Services

Marion PODESTA* Jean-Christophe POUDOU†

November 17, 2011

Abstract

This paper analyzes optimal nonlinear pricing for a monopoly supplying a bundle of a commodity and a related service, where the consumers' private information is captured by scalar variable. With constant marginal costs, we find the standard result where the good and the service in the bundle are lower than separately. However, under the increasing cost assumption, when the degree of the complementarity becomes sufficiently high, the marginal price of separate good is lower than the good price in the bundle. Contrary to Martimort (1992), when good and service are perfectly complementary, we can not conclude that it is costly for consumers to sign two contracts from different shops than to buy the bundle. Because of asymmetric properties in the utility function, profitability result of bundling strategy depends, on the one hand, on the degree of complementarity between commodity and related service and on the other hand, on the degree of the optional service.

JEL Classification: D42, L12, Q4

Keywords: Bundling, Nonlinear pricing, Energy market

Résumé

Cet article analyse la tarification non linéaire optimale pour un monopole offrant un package comprenant un bien et un service attaché au bien, lorsque ce dernier augmente l'utilité marginale que les consommateurs accordent au bien. Nous examinons les mécanismes d'incitation de révélation des préférences dans le cas où l'information privée est représentée par un paramètre unidimensionnel. Sous l'hypothèse de coûts constants nous retrouvons le résultat standard où il est profitable pour les consommateurs d'acheter le bien et le service attaché sous forme de package plutôt que séparément. Cependant, avec des coûts croissants, lorsque le degré de complémentarité entre le bien et le service est suffisamment élevé, le prix marginal du bien séparé est plus faible que comme élément du package. Contrairement à Martimort (1992), lorsque le bien et le service sont parfaitement complémentaires, nous ne pouvons pas conclure qu'il est plus coûteux pour les consommateurs de signer deux contrats aux différents magasins que d'acquérir le package. Du fait des

*Corresponding author: marion.podesta@univ-savoie.fr, IREGE, University of Savoie, 4 Chemin de Bellevue, BP 80439 Annecy-le-Vieux.

†LAMETA, University of Montpellier I.

asymétries prises en compte dans la fonction d'utilité, la profitabilité d'une stratégie de ventes liées dépend, d'une part du degré de complémentarité entre le bien et le service attaché, et d'autre part du degré d'optionalité du service lié.

JEL classification: D42, L12, Q4.

Mots-clés : Ventes liées, Tarification non-linéaire, Marchés énergétiques

1 Introduction

Recently, a trend towards deregulation of utilities industries (as telecommunications or energy sectors) has been observed worldwide and has an impact on market structure. This new environment incites firms to diversify their offers, and they can compete with their new rivals with the help of bundles. Simultaneously, a convergence phenonenom appears and creates a strategic link between different markets. In telecommunications sectors, the process of convergence across multimedia and telecom markets involves *triple-play* or *quadruple-play* offers. These packages include broadband, TV and fixed telephony at bundle discounts. In energy markets incumbents supply bi-energy bundles or offers including energy and services.[1] Subsidiaries in telecommunications and energy sectors allow to incumbents to propose bundles with several goods or services, therefore incumbents keep a dominant position in these markets.

This paper deals with a private monopoly, mainly to depict a situation where market power is high. We study pricing strategy for a firm supplying a good and a complementary optional service, when the service increases the good's marginal utility and where the consumers' private information is captured by scalar variable.[2] In our model, we use the analysis of Martimort (1992, 1996) in a principal-agent context. Firm can sell good and service separately (independent pricing) or as a bundle (pure bundling[3]). Contrary to the Martimort's assumptions, there is no symmetry between the good and the related service in the consumers' utility function. In order to study the specificity of the service, we use the nonlinear pricing. More precisely, the paper focuses on bundling to satisfy a fundamental need (space heating needs, for instance), where the good can be tied with a related service.[4] We show that the profitability of bundling depends on the degree of complementarity between the good and the service, as well as on the optional character of the service.

Martimort (1992, 1996), in a principal-agent model, introduces the possibility for a common agent to contract with multiple principals. He compares the cooperative situation, this is the case where firms can offer a bundle, and the situation with noncooperation under the assumption of nonlinear pricing. The results depend on the complementarity

[1] For example, Gaz de France-Suez proposes "Provalys" to professionals and "DolceVita" to residential consumers. In parallel, Electricité de France provides "Essentiel Pro" to professionals and "Bleu Ciel" to residential consumers. These bundles include energy and services as optimization of energy consumption. For more details see http://www.energieetservice.fr/energie-et-service.

[2] It is most often assumed that the firm can observe only one variable. It is also common to suppose that the observed variable has a single dimension, this is quality in Mussa and Rosen (1978) and quantity in Maskin and Riley (1984). Wilson (1993) provides definitions and examples of multidimensional goods and pricing.

[3] Pure bundling refers to the practice of selling two or more goods and/or services together in a bundle at a unique price.

[4] The service for example may be technical maintenance or energy consultancy which enhances the gross utility for the good.

or the substituability between activities controlled by each principal. If the goods are complements, bundling is an optimal strategy for the principals and consumers. However, if the goods are substitutes, the consumers' utility is better when there is noncooperation between the principals, but profits are lower.

The bundling literature in the monopoly case shows that the optimal strategy is the mixed bundling one for it obtains a maximum surplus from the consumers if the correlation of reservation values is negative (Adams and Yellen, 1976, Schmalensee, 1984 and Mc Afee et al, 1989). With the mixed bundling strategy, the goods are available both separately and in a bundle. Since bundling allows the monopolist to extract more consumer rent, it makes higher profits (since the correlation of consumers' reservation values is negative). However when competition increases, results are reversed. Bundling can reduce the firms profit and increase the consumers' rent.[5] Literature on bundling has not been really explored bundling strategy for a complementary service. In fact, models rarely integrate the specificity of the service into the pricing strategy. The acknowledged fact is that when the service is personalized, its ex-ante valuation is difficult (Bateson, 1995). In the general case, the consumers' commodity value is better than the service value. Thus, a consumer has more difficult to give a benchmark price for a service, relatively to a good. When monopoly uses a bundling strategy, it has an additional tool to practice price discrimination. Moreover, nonlinear pricing is widely used in several markets (energy or telecommunication markets, for instance) to supply one or several goods.[6] Firm proposes a menu of tariff to incite consumers to reveal their preferences[7] and can extract more consumers' surplus.

This paper considers the case where the monopoly practices nonlinear pricing for both a good and an attached service. The principal (monopoly) has two shops which can offer the commodity and the optional service. We compare marginal prices for the good and the service under an independent pricing strategy and when bundling is considered. In the bundling case, both shops coordinate their pricing strategy to provide a bundle. In the paper, two cases are considered. Firstly, monopoly costs are constants, the two shops are supposed to be only retailers. Secondly, we consider that the different shops produce the good and the service under increasing costs. However, incomplete information on consumers' type implies to implement an incentive compatible nonlinear pricing schedule in both cases. Our definition of bundling is slightly different from the standard industrial organization literature since we aim to study informational aspects of this kind of strategy by a monopolist. Here, we consider that two productive units (or shops) can separately produce the core good and the attached service but the commercial unit could sold either separately or together. It is thus in this business view that we study the informational gains and losses and the effects of these practices on prices.

Under constant costs assumption, we show that it is less costly for consumers to buy the bundle and firm has lower consumers' rent compared with the independent pricing strategy. Indeed when monopoly follows an independent pricing strategy, it would be worse off to propose separating contracts. The monopoly proposes a contract for the commodity and another one for the service, it can capture more consumers' rent from the signature of each contract. However, when the monopoly provides a bundle it cannot duplicate the independent contracts and captures only once the consumers' rent.

With increasing costs and when the good and the service are perfectly complementary,

[5]See Reisinger (2006) and Economides (1993) in the duopoly case.

[6]For example, for the gas supply, the industrial gas retailler uses a two-part tariff. It comprises a uniform price for each unit of gas purchased plus a fixed fee payable if any positive amount is purchased.

[7]Since consumers' willingness to pay is a private information, a problem of asymmetric information appears between firm and consumers. These relations are described in principal-agent models.

the optionality of the service has an impact on the marginal price of separate good. The marginal price of the good is lower separately than as a component of the bundle. Because of asymmetries in the utility function, the profitability of bundling depends on the trade-off between the degree of complementarity and the degree of the optional service.

The following section sets out the model. Firstly, we consider that the monopoly has constant production costs. The section 3 considers the case where the shops coordinate their offers and propose a bundle, and in section 4, we remove this assumption to consider that the two shops provide the commodity and the service separately. The section 5 compares the different types of strategies. Secondly, in order not to limit our analysis at constant costs and to focus on the energy retailers concerns, we discuss to increasing costs in section 6. Finally, section 7 proposes few concluding remarks.

2 The model

The model focuses on energy demand to satisfy a fundamental need, for instance space heating, lighting or air conditioning needs for residential consumers. We consider a space heating need for instance, which can be provided by a couple of energy (x), gas or electricity and also a level of service (s). The firm offers a good (energy) and a related (complementary) service, it can be technical maintenance or energy consultancy. The gross utility is given by:

$$u(H, \theta)$$

where H is the level of heating achieved and depends on the consumers' preferences denoted θ. Heating is obtained by a given technology which combined both the quantity of energy consumed $x \geq 0$, and the related service level purchased $s \geq 0$. Hence, the utility can be directly written as an increasing asymmetric function of x and s, let $u(H, \theta) = u(x, s, \theta)$ which satisfies the following requirements for all $y \geq 0$:

$$\forall y, u(y, 0, \theta) \geq u(0, y, \theta) \tag{H1}$$

Moreover the usual Spence-Mirless property is verified and given by:

$$u_{\theta x}(x, s, \theta) > u_{\theta s}(x, s, \theta) \geq 0 \tag{H2}$$

where subscripts represent partial second derivatives. The (energetic) good is intrinsically preferred to the service since the component cannot provide heating alone. As a result, the technical service can be viewed as optional from the consumer's point of view as its marginal utility is always smaller than the one for the good for all types considered. Whenever $u_{\theta x}(x, s, \theta) > u_{\theta s}(x, s, \theta)$, the service is not totally an option but neither essential. However if it is the case that $u_{\theta s}(x, s, \theta) = 0$, the service is viewed as a pure option from the consumer's point of view.

We also assume strict concavity[8] and complementarity between energy and service such as:

$$u_{xs}(x, s, \theta) \geq 0 \tag{H3}$$

The preferences of consumers are given by a single-dimensional parameter $\theta \in \Theta \equiv [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$ with a distribution function $F(\theta)$.[9] In incomplete information, the monopoly

[8] More precisely this implies that $u_{ss} < 0, u_{xx} < 0$ and $u_{ss}u_{xx} - (u_{xx})^2 > 0$. Moreover we assume $u_{\theta xx}(x, s, \theta) > 0$.

[9] The density function $f(\theta)$ is supposed to be everywhere non-negative.

knows the distribution of consumers' preference but it doesn't know exactly each consumer's reservation price. We also define $\varphi(\theta) = \frac{1-F(\theta)}{f(\theta)}$, the *informational cost or virtual cost*, which represents a cost for the firm, and it is decreasing[10] in θ: $\varphi'(\theta) \leq 0$. The information cost is generated by the informational advantage of the agent over the principal.

To simplify, the costs are supposed to be identical and increasing convex for both service and good units produced:

$$C(x,s) = w(x+s) + \frac{k}{2}(x^2+s^2)$$

In order not to limit our study, we analyze two normalized cases. Firstly, we suppose that the shops sell their products at the market prices, in this case we suppose that marginal costs are constant. The linear cost function (with $k=0$), when the monopoly is a simply retailer of the energy and the service, is given by:

$$C(x,s) = w(x+s) \tag{H4}$$

with $c(x) = C(x,0)$ and $c(s) = C(0,s)$ are respectively the cost function of the good and the service.

Secondly, we remove this assumption to focus on the case where the shops produce energy and service sold to consumers, i.e when costs express increasing marginal slope so we normalize $w = 0$. The following increasing convex function satisfies the previous assumptions:

$$C(x,s) = \frac{k}{2}(x^2+s^2) \tag{H5}$$

In order to simplify the analysis and without loss of generality, we set $k=1$.

Suppose that the monopolist sells its products using a nonlinear tariff $\Im$, the consumers' net utility is given by:

$$U = \begin{cases} u(x,s,\theta) - \Im & \text{if } x,s > 0 \\ 0 & \text{if } x,s = 0 \end{cases}$$

where $\Im$ is the total expenditure paid if consumers buy either x units of good and s units of service in the bundling case, either x, or s in the independent pricing case. With the independent pricing strategy, $\Im$ can be divided in two respective parts, t for the energy and τ for the related service: $\Im = t + \tau$. With bundling strategy, the nonlinear tariff is denoted $\Im = T$. In each case, we consider nonlinear tariffs with a fixed fee and a variable part.

The timing of the game is the following. First, the agent discovers his type. Then, the principals offer simultaneously the contracts, or equivalently the direct revelation mechanisms: $T(\hat{\theta})$ or $t(\hat{\theta})$ and $\tau(\hat{\theta})$. Second, the agents accept or refuse the proposal they respectively receive. If the agents accept, they make their reports to the principal. Finally, the tariff $T(\hat{\theta})$ or $t(\hat{\theta})$ and $\tau(\hat{\theta})$ as well as the quantities of good and attached service $\{x(\hat{\theta}), s(\hat{\theta})\}$ are implemented.

In our setting, the firm only cares about its expected profits, and seeks to maximize $E(\pi)$. Moreover, the consumer must have adequate incentives to reveal his type

[10] This is an acceptable assumption as Bagnoli and Bergstrom (2005) have shown. As the type increases, the relative weight of types above θ decreases. The firm is more concerned about the costs left below θ.

truthfully—incentive compatibility (IC)—and to participate in the mechanism voluntary—individual rationality (IR). The program facing the monopoly is:

$$\begin{aligned} \max E(\pi) &= \int_{\underline{\theta}}^{\bar{\theta}} \{\Im(\theta) - C(x(\theta), s(\theta))\} f(\theta) d\theta && \text{(EP)} \\ U(\theta) &\equiv u(x(\theta), s(\theta), \theta) - \Im(\theta) \geq u(x(\hat{\theta}), s(\hat{\theta}), \theta) - \Im(\hat{\theta}) && \text{(IC)} \\ U(\underline{\theta}) &\geq 0 && \text{(IR)} \end{aligned}$$

The(IC) constraint means that the firm lets a consumers' surplus higher when the consumer reveals his preferences and chooses an optimal price schedule than when he lies about his preferences and would look like another type. With this constraint the monopoly encourages the consumers to reveal their preferences. With the (IR) constraint the firm must incite the consumers to purchase by leaving them a positive net surplus.

To go further, we can specify a quasi-concave function for which assumptions above are fulfilled:

$$u(x, s) = \theta(x + \alpha s) + 2\beta xs - \frac{\gamma}{2}x^2 - \frac{\gamma}{2}s^2 \tag{H6}$$

where $\alpha \in [0, 1]$; $\beta \in [0, \frac{1}{2}]$ and without loss of generality, we set $\gamma = 1$. The parameter α represents the optional character of the service. If $\alpha = 0$, it is viewed as totally optional and respectively when $\alpha = 1$, it becomes essential to satisfy the fundamental need. The parameter β is the degree of complementarity between the good and the attached service. If $\beta = 0$, they are independent and contrary if $\beta = \frac{1}{2}$, the good and the service are perfectly complementary.

The information cost $\varphi(\theta)$, previously defined, can be specified with a uniform law as:

$$F(\theta) = \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \text{ thus } \varphi(\theta) = \bar{\theta} - \theta \tag{H7}$$

We will use this assumption in the sequel in particular to determine an implicit solution in the independent pricing case.

In the following, we will focus on interior allocation for good and service $(x, s > 0)$, we will explicit the parametric restrictions involved when necessary. In sections 3 to 5, we first consider that (H4) is true, that is costs are constants and we relax this assumption in section 6, assuming that (H5) holds..

3 Nonlinear pricing schedule and bundling

In this section, the monopoly can offer a bundle composed of a good (energy) and a related complementary service (for instance technical consultancy). The monopoly has two separated shops which coordinate to propose a nonlinear tariff $\Im = T$, for the bundle.

Ex-post, the nonlinear tariff T is implemented as a three-part schedule and has the following form:

$$T = Z + px + rs \Leftrightarrow Z = T - px - rs$$

where Z is the fixed fee of the tariff, p and r are respectively the energy and the service rates, and x and s are the relative quantities. Solving the restated problem (see Appendix B for details), the quantities of good and service are given by:

$$x^B(\theta) = \frac{(\theta - \varphi(\theta))(1 + 2\beta\alpha) - w(1 + 2\beta)}{(1 - 4\beta^2)} \text{ and } s^B(\theta) = \frac{(\theta - \varphi(\theta))(\alpha + 2\beta) - w(1 + 2\beta)}{(1 - 4\beta^2)}$$

The quantities[11] increase with consumers' willingness to pay.[12] The degree of optionality has a more important impact for the service in the bundle than for the good and this as much as the degree of complementarity is higher.[13] Therefore, T is the overall tariff that consumers pay to purchase the bundle. The fixed fee of the three-part tariff can be directly restated as: $Z(\theta) = T(\theta) - p^B(\theta)x^B(\theta) - r^B(\theta)s^B(\theta)$.

Proposition 1 *The marginal price of the service depends on the degree of optionality but it is not the case for the commodity price, these prices are given by:*

$$\begin{aligned} p^B(\theta) &= p^F(\theta) + \varphi(\theta) \\ r^B(\theta) &= r^F(\theta) + \varphi(\theta)\ \alpha \end{aligned}$$

Here $p^F(\theta)$ and $r^F(\theta)$ are respectively prices of good and related service in first-best situation.[14] The marginal price of the service is lower than the marginal price of the good because the service is optional. The monopoly less distorts the service price in relation to the good price and this effect is due to the asymmetric information. As marginal price of the service is low, as well as its quantity, the information cost is captured by the fixed fee of the three part schedule.[15] Concerning good price in the bundle, it not depends on the optional character of the service.

In a more general setting, at the equilibrium, prices of commodity and service would verify (omitting arguments):

$$\begin{aligned} p^B(\theta) &= u_x(x^B, s^B, \theta) = c'(x^B) + \varphi(\theta)u_{\theta x}(x^B, s^B, \theta) \\ r^B(\theta) &= u_s(x^B, s^B, \theta) = c'(s^B) + \varphi(\theta)u_{\theta s}(x^B, s^B, \theta) \end{aligned}$$

There are two effects for marginal prices in bundle. First, marginal prices are higher than in complete information due to the information cost which is positive. Second, quantities are lower than in complete information situation. Indeed in this general setting, it is difficult to measure which effect dominates. Proposition 1 clarifies these tradeoff in our specific case.

4 Separate sales and nonlinear tariff

In this section, we consider the situation where two principals (the shop 1 is the energy retailer and the shop 2 is the service retailer) supply their contracts to a same type of agent under a nonlinear pricing defined by: $\Im = t + \tau$. However, our analysis slightly differs from Martimort (1992) as $u(x, s, \theta)$ cannot be a symmetric function since energy and service don't fulfilled exactly the same intrinsic needs.

The agent maximizes his utility in relation to the rates of the good and the service bought and in relation to his report type $(\hat{\theta})$ according his own type (θ). At equilibrium the agent report is truthfully, so the utility function for a consumer is given by:

$$U(\theta) = \max_{\hat{\theta}_x, \hat{\theta}_s} u(x(\hat{\theta}_x), s(\hat{\theta}_s), \theta) - t(\hat{\theta}_x) - \tau(\hat{\theta}_s) \text{ when } (\hat{\theta}_x, \hat{\theta}_s) = (\theta, \theta)$$

[11] For a positive level of attached service, we assume a condition on w i.e. $w < \bar{w}$ where $\bar{w} = \frac{\underline{\theta}(\alpha+2\beta) - \Delta\theta(\alpha+2\beta)}{1+2\beta}$.

[12] See appendix B.1.

[13] Indeed, $\frac{dx^B}{d\alpha} = \frac{2\beta(\theta - \varphi(\theta))}{1-4\beta^2} < \frac{ds^B}{d\alpha} = \frac{(\theta - \varphi(\theta))}{1-4\beta^2}$.

[14] These levels are detailed in the Appendix A.

[15] Here $\frac{dZ}{d\alpha}(\bar{\theta}) = \bar{\theta}s^F(\bar{\theta}) > 0$. The fixed fee of the three part schedule Z is increased in α for $\bar{\theta}$.

Here t is the price for the energy bought from the shop 1 and τ is the price for the level of service purchased from the shop 2. *Ex-post,* the reports for energy and service are truthfully: $(\hat{\theta}_x = \theta)$ and $(\hat{\theta}_s = \theta)$. The first order of incentive compatibility constraints are given by:

$$(\hat{\theta} = \theta) \begin{cases} u_x(x(\theta), s(\theta), \theta)\dot{x}(\theta) - \dot{t}(\theta) = 0 \\ u_s(x(\theta), s(\theta), \theta)\dot{s}(\theta) - \dot{\tau}(\theta) = 0 \end{cases} \tag{1}$$

The necessary optimal condition can allow us to write (with the envelope theorem):

$$\dot{U}(\theta) = u(x(\theta), s(\theta), \theta)$$

This rent is increasing and must keep a positive value for all values of θ (under the IR constraint). Consequently, we minimize the consumer rent with low bound constraint in $\underline{\theta} : U(\underline{\theta}) = 0$.

The principals, the energy retailer (shop 1) and the service retailer (shop 2), must implemented a mechanism design which incite the consumer to tell the true and to reveal their preferences. *Ex-post,* the agent would be well advised to report his true type than he lies about their preferences $(\hat{\theta}_x, \hat{\theta}_s) = (\theta, \theta)$. The second order incentive conditions are given by the sign of the hessian of U and are given in appendix C.1.

4.1 Sales of good

In this case we consider only the program of the shop 1. *Ex-post,* the two-part tariff for the energy supply is:

$$t = A + px \Leftrightarrow A = t - px$$

where A is the fixed fee of the tariff and t is the overall price that consumers pay from energy purchase. The cost, with an independent pricing, is proportionate to the quantity of good bought and with constant costs (H4) is given by: $C(x, 0) = c(x)$. The shop 1 maximizes its expected profits (EP) under the first and the second order incentive constraints (1) with $t(\theta) = t(x(\theta))$. In appendix C.1 using the analysis of Martimort (1992) we solve the shop 1 profit maximization problem. In general, if we note $\varphi(\theta) = \frac{1-F(\theta)}{f(\theta)}$, the equilibrium price for the good is given by:

$$p^{IP}(\theta) = c'(x^{IP}(\theta)) + \varphi(\theta)\left(u_{\theta x}(x^{IP}(\theta), s^{IP}(\theta), \theta) + I^{IP}(\theta)\right) \tag{2}$$

Comparing this price[16] with the energy price in the bundling case, the additional term $I^{IP}(\theta)$ is positive thus it increases the marginal price of the separate good. Nevertheless, this term $I^{IP}(\theta)$ is not influenced by the optionality of the service. In the separate sales, we can conclude that the independent good price is always higher than as component of the bundle.

4.2 Sales of service

For the sales of service, we consider only the program of the shop 2. As for the shop 1 in the previous section, we give the *ex-post* two-part tariff for the service supplied:

$$\tau = B + rs \Leftrightarrow B = \tau - rs$$

[16] With $x(\overline{\theta}) = x^F(\overline{\theta})$ and $I^{IP}(\theta) = \dfrac{u_{\theta s}(x(\theta), s(\theta), \theta)u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + u_{\theta s}(x(\theta), s(\theta), \theta)}$.

where B is the fixed fee of the tariff and τ is the overall price that consumers pay for the service bought. With (H4), costs are proportionate to the level of service chosen and are given by: $C(0, s) = c(s)$.

In appendix C.2, we solve the shop 2 profit maximization problem. the equilibrium price of service is given by:

$$r^{IP}(\theta) = c'(s^{IP}(\theta)) + \varphi(\theta) \left(u_{\theta s}(x^{IP}(\theta), s^{IP}(\theta), \theta) + J^{IP}(\theta) \right) \tag{3}$$

Comparing with the marginal price of service[17] as part of bundle, the term $J^{IP}(\theta)$ increases the separate marginal price since it is positive. However, as for the good sales, this additional term $J^{IP}(\theta)$ is not influenced by the optionality of the service. In the following section, we solve the system.

4.3 Independent pricing schedule

The contract is optimal if there is a couple $\{x^*(\theta), s^*(\theta)\}$ which satisfy the equation system (2) and (3), so called Hamilton-Jacobi system; -omitting arguments:

$$\left\{ \begin{array}{l} -c'(x) + u_x(x, s, \theta) - \varphi(\theta) \left(u_{\theta x}(x, s, \theta) + I^{IP}(\theta) \right) = 0 \\ -c'(s) + u_s(x, s, \theta) - \varphi(\theta) \left(u_{\theta s}(x, s, \theta) + J^{IP}(\theta) \right) = 0 \end{array} \right. \tag{4}$$

with $x(\bar{\theta}) = x^F(\bar{\theta})$ and $s(\bar{\theta}) = s^F(\bar{\theta})$. In general, one can conclude that the separate marginal prices of the good and the service are always higher than as a part of bundle. As Martimort (1992), one find that it is costly for consumers to buy the good and the service separately due to the implicit competition between the two shops. In the general case, bundling is an optimal strategy for both principals and consumers.

In our specific setting (with constant costs), the Hamilton-Jacobi system (4) can be restated, as -omitting arguments-:

$$\left\{ \begin{array}{l} (2\theta - \bar{\theta}) - w - x + 2\beta s - (\bar{\theta} - \theta) \frac{2\alpha\beta \ \dot{s}}{2\beta \ \dot{x} + \alpha} = 0 \\ (2\theta - \bar{\theta})\alpha - w - s + 2\beta x - (\bar{\theta} - \theta) \frac{2\beta \ \dot{x}}{2\beta \ \dot{s} + 1} = 0 \end{array} \right.$$

Using the method of Olsen-Osmundsen (2001) and under specification (H7), there is an optimal linear solution $\{\eta_x, \eta_s\}$ which allow us to rewrite the program as:

$$\begin{array}{rcl} x^{IP}(\theta) & = & x^F(\theta) + (\bar{\theta} - \theta)\eta_x \\ s^{IP}(\theta) & = & s^F(\theta) + (\bar{\theta} - \theta)\eta_s \end{array} \tag{5}$$

Where $x^F(\theta)$ and $s^F(\theta)$ are respectively quantities of good and service.[18] At the equilibrium, values of η_x and η_s given in the appendix C.3 are negative, one can directly conclude that these quantities are sub-optimal than in the first-best situation. The marginal prices are given by:

$$\begin{array}{rcl} p^{IP}(\theta) & = & p^F(\theta) + (\bar{\theta} - \theta)(1 + \alpha Y) \\ r^{IP}(\theta) & = & r^F(\theta) + (\bar{\theta} - \theta)(\alpha + Y) \end{array}$$

[17] With $s(\bar{\theta}) = s^F(\bar{\theta})$ and $J^{IP}(\theta) = \dfrac{u_{\theta x}(x(\theta), s(\theta), \theta) u_{xs}(x(\theta), s(\theta), \theta) \dot{x}(\theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta) + u_{\theta x}(x(\theta), s(\theta), \theta))}$.

[18] As in the bundling situation, we give the condition on w for which the level of attached service is positive: $\tilde{w} = \frac{\Delta\theta - \Delta\theta(2\beta(4\beta - \alpha) + \sqrt{1 + 32\,\beta^2}(1 + 2\beta\alpha)) + 4\beta\underline{\theta}(\alpha + 2\beta)}{4\beta(1 + 2\beta)} > w$.

with $Y = \frac{y}{X}$ and $y = 16\beta^2 + \sqrt{1 + 32\,\beta^2} - 1$ (here $Y \geq 1$). These prices are both increasing with the optional character of the service.[19] Nevertheless, the parameter α affects more the information cost for the good than for the service.[20]

5 Bundling strategy versus independent pricing

In this section, we compare marginal prices when shops coordinate on a bundling strategy and the case where there is no cooperation with an independent pricing strategy. At the equilibrium, the quantities[21], under our specific assumptions, can be rewritten as:

$$\begin{aligned} x^F(\theta) &> x^B(\theta) > x^{IP}(\theta) \\ s^F(\theta) &> s^B(\theta) > s^{IP}(\theta) \end{aligned}$$

In the bundling case, the optimal quantities of good and service are higher than in the separate sales. In the explicit case, we can rewrite marginal prices of the energy and the related service as:

$$\begin{aligned} p^{IP}(\theta) &= p^B(\theta) + (\bar{\theta} - \theta)\alpha Y \\ r^{IP}(\theta) &= r^B(\theta) + (\bar{\theta} - \theta)Y \end{aligned} \tag{6}$$

It implies that the marginal prices are higher when shops provide two different contracts than when they provide a bundle (since $Y > 0$). In the section 3, we have shown that the marginal price of the service in the bundle depends on the degree of its optionality. With an independent pricing strategy, the degree of the optional service has also an impact on the marginal price of good. The parameter α has an influence on the information cost for the separate good.

When service is purely optional ($\alpha = 0$), the marginal price of good is identical in an independent pricing strategy or in a bundling one, that is for all $\theta \in \Theta$, but it is not necessarily the case for the service rate. Indeed, the marginal price of service is always higher independently than in a bundle. As the service is not essential to satisfy the fundamental need, the monopoly fixes a lower price for the service in a bundle than independently and raises the price of good as a component of the bundle.

Proposition 2 *One can conclude that:*

$$\begin{aligned} p^F(\theta) &< p^B(\theta) \leq p^{IP}(\theta) \\ r^F(\theta) &\leq r^B(\theta) < r^{IP}(\theta) \end{aligned}$$

The price gap between the energy price in a bundle and sold separately depends on the degree of the optional service.

The previous proposition states that the optional characteristic has an impact on the marginal prices of the independent good and the service bound. The discount when consumers purchase the good in the bundle rather than separately is informational (see

[19] Indeed, $\frac{d(1+\frac{\alpha y}{X})}{d\alpha} = \frac{y}{X}$ and $\frac{d(\alpha+\frac{\alpha y}{X})}{d\alpha} = 1$.
[20] Indeed, $\frac{dp^{IP}}{d\alpha} = y\varphi(\theta) \geq \frac{dr^{IP}}{d\alpha} = \varphi(\theta)$.
[21] See appendix D.1.

equation 6). Whereas, this informational rebate appears for the service when it is close to the first-best situation.

These results come from the asymmetric utility function (assumptions H1 and H2), since energy and service don't fulfilled exactly the same intrinsic needs. The independent pricing strategy implies inefficiencies due to the implicit competition between the two shops, the principals should coordinate to provide a bundle.

The global analysis is done in Martimort (1992, 1996) in the case where $u(\cdot)$ has strong symmetric properties. If it was the case here then any optimal independent nonlinear pricing scheme leads a continuum of allocations $(x^{IP}(\theta), s^{IP}(\theta))$ such that[22]:

$$s^{\infty}(\theta) = x^{\infty}(\theta) \leq x^{IP}(\theta) = s^{IP}(\theta) \leq x^{B}(\theta) = s^{B}(\theta) \tag{7}$$

In this symmetric setting, (which would correspond to our limit case where $\alpha = 1$), Martimort (1992) has shown that independent pricing introduces inefficiencies due to the implicit competition between shop managers (principals) and of course bundling is dominant ex ante, that is $E(\pi^B) \geq E(\pi^I)$, since by definition bundling maximizes the total expected profit. In our model (where $\alpha \leq 1$), one cannot directly conclude that the inequality (7) holds because of asymmetric properties (H1)-(H3) we consider. The optimal quantities of the good and the service when shops follow an independent pricing strategy are smaller than the cooperative situation that is:

$$s^{\infty}(\theta) = x^{\infty}(\theta) \leq s^{IP}(\theta) \leq x^{IP}(\theta) < x^{B}(\theta) = s^{B}(\theta)$$

Concerning marginal prices, in the specific case, one can conclude that:

$$p^{F}(\theta) = r^{F}(\theta) < r^{B}(\theta) \leq p^{B}(\theta) \lesseqgtr r^{IP}(\theta) \leq p^{IP}(\theta)$$

In the benchmark, the marginal prices of good and service are equal, as well in the bundle when the service is considered as mandatory ($\alpha = 1$). Moreover, prices are both lower in the benchmark than as components of the bundle. The information cost tends to increase rates in incomplete information case.

However, it exists a threshold of complementarity degree between good and service: $\beta^T(\theta) = \frac{1}{2}\frac{\alpha-1}{\alpha^2-2\alpha-1}$, where the marginal price of the good in bundle can be higher than the level of separate service ($\beta < \beta^T(\theta)$). Finally, marginal price of the separate service is always lower than price of the separate good unless they are perfectly complementarity (if $\beta = \frac{1}{2}$).

In order not to limit our analysis at constant costs and to focus the energy retailers concerns, the following section introduces the increasing costs assumption.

6 Increasing marginal costs

Until now, we have considered the case where the different shops are only retailers. In this section, we remove this assumption to consider the situation where the two shops produce good (energy) and service sold to consumers. In this way, contrary to the constant costs with market prices, principals have increasing costs, given by (H5) in section 2, introducing decreasing scale returns.

[22] Solution $s^{\infty}(\theta) = x^{\infty}(\theta)$ are such that:

$$-c'(x) + u_x(x, x, \theta) - 2\varphi(\theta) u_{\theta x}(x, x, \theta) = 0$$

6.1 Bundling strategy

In the bundling strategy, the shops 1 and 2 can provide a bundle. At the equilibrium, quantities of good and service are given by:

$$x^B(\theta) = x^F(\theta) - (\bar{\theta} - \theta)\frac{1 + \alpha\beta}{2(1-\beta^2)} \text{ and } s^B(\theta) = s^F(\theta) - (\bar{\theta} - \theta)\frac{\alpha + \beta}{2(1-\beta^2)} \tag{8}$$

where $x^F(\theta)$ and $s^F(\theta)$ are still quantities of good and attached service in the first-best situation (given in Appendix A). These quantities are increasing[23] with the type of consumers. Moreover, the impact of optionality degree is more important for the service bought than for the good,[24] and this as much as the degree of complementarity is high.

When the two shops follow a pure bundling strategy, they propose a unique nonlinear tariff for the bundle. Marginal prices of the good and the service, are given by:

$$\begin{aligned} p^B(\theta) &= p^F(\theta) + (\bar{\theta} - \theta)\frac{(1 - \beta\alpha - 2\beta^2)}{2(1-\beta^2)} \\ r^B(\theta) &= r^F(\theta) + (\bar{\theta} - \theta)\frac{(\alpha - \beta - 2\alpha\beta^2)}{2(1-\beta^2)} \end{aligned}$$

As $\beta \in [0, \frac{1}{2}]$ and $\alpha \in [0, 1]$, the energy price is always higher in the bundle than in the complete information situation due to the information cost which is positive, even if the service is purely optional.

As well as in the bundling situation with constant costs, the effect of the optional service increases marginal prices but this impact is more pronounced for the level of service and when the degree of complementarity is high.[25] The information cost is low for the marginal price of service on account of the optional character.[26] Therefore, the service is less costly than the good in the bundle,[27] this is due to the fact that consumers enjoy a higher utility by consuming energy relatively to the service.

6.2 Independent pricing strategy

Now the system (4) can be rewritten as:

$$\begin{cases} 2\theta - \bar{\theta} - 2x + 2\beta s - (\bar{\theta} - \theta)\frac{2\alpha\beta\ \dot{s}}{2\beta\ \dot{x} + \alpha} = 0 \\ (2\theta - \bar{\theta})\alpha - 2s + 2\beta x - (\bar{\theta} - \theta)\frac{2\beta\ \dot{x}}{2\beta\ \dot{s} + 1} = 0 \end{cases}$$

The method used is the one of Olsen-Osmundsen (2001), as in section 4. With an uniform law, it exists a linear optimal solution $\{\kappa_x, \kappa_s\}$ such that:

$$\begin{aligned} x^{IP}(\theta) &= x^F(\theta) + (\bar{\theta} - \theta)\kappa_x \\ s^{IP}(\theta) &= s^F(\theta) + (\bar{\theta} - \theta)\kappa_s \end{aligned}$$

[23] See appendix B.2.

[24] Indeed: $\frac{dx^B}{d\alpha} = \frac{\beta(\theta - \varphi(\theta))}{2(1-\beta^2)} < \frac{ds^B}{d\alpha} = \frac{(\theta - \varphi(\theta))}{2(1-\beta^2)}$.

[25] Indeed: $\frac{dp^B}{d\alpha} = \frac{\beta(\theta - \varphi(\theta))}{2(1-\beta^2)} < \frac{dr^B}{d\alpha} = \frac{\theta - \varphi(\theta)(1-2\beta^2)}{2(1-\beta^2)}$.

[26] Here $(1 - \beta\alpha - 2\beta^2) - (\alpha - \beta - 2\alpha\beta^2) = (2\beta + 1)(\beta - 1)(\alpha - 1) > 0$

[27] Indeed: $(p^B - r^B) = \frac{(-\alpha+1)(-2\beta\varphi(\theta) - \varphi(\theta) - \theta)}{2(\beta+1)} > 0$.

At the equilibrium, the couple $\{\kappa_x^*, \kappa_s^*\}$ is given in appendix C.4 and values of parameters κ_x and κ_s are both negative. The marginal prices are given by:

$$\begin{aligned} p^{IP}(\theta) &= p^F(\theta) + \frac{1}{D}(\bar{\theta} - \theta)(\alpha m + n) \\ r^{IP}(\theta) &= r^F(\theta) + \frac{1}{D}(\bar{\theta} - \theta)(m + \alpha n) \end{aligned}$$

we show that[28] $m > 0 \geq n$. These prices depend on the degree of complementarity as well as the degree of the optional service. Since $m > n$, the parameter of optionality α affects the information cost and more on marginal price of service than on the marginal price of good. The introduction of increasing costs has an impact on the good price contrary to section 4. Here the monopoly less distorts the good price than the service price.

6.3 Bundling versus independent pricing strategies

The figure 1 compares bundling and independent pricing strategies with increasing costs. Results differ from those of section 5 under the constant costs assumption.

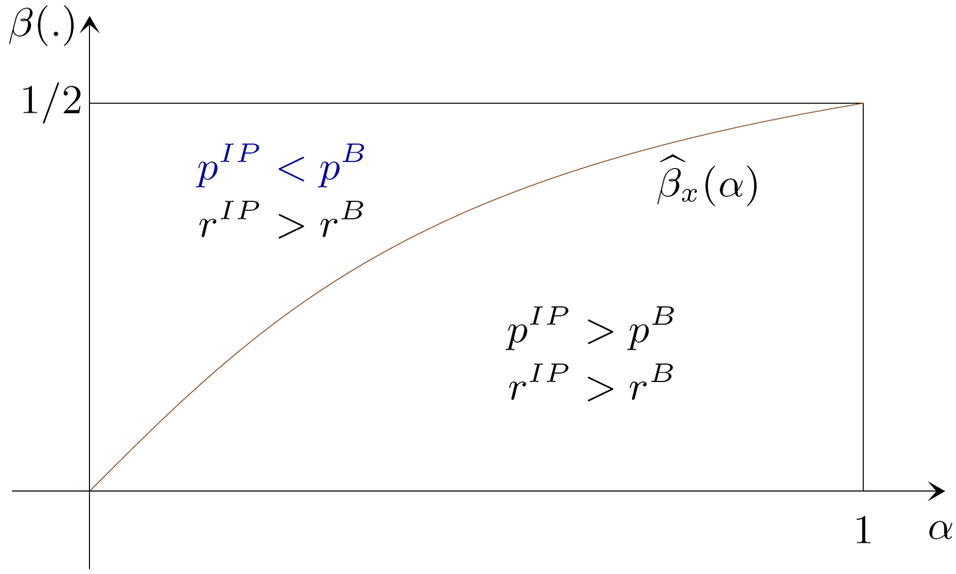

Figure 1: Comparison between prices of good and service independently and in a bundle

There are two areas in the figure 3. In the first area, below to the full line curve (where $\beta < \hat{\beta}_x(\alpha)$), the separate prices for the good and for the service are both higher than those as components of the bundle. This situation is the standard case where a consumer yields a discount for the bundle rather than the separates sales. This analysis holds when the good and the service are independent, where $\beta = 0$. Usually the conventional wisdom (as

[28] with $m = 8\beta^4 - 3(\sqrt{8\beta^2+1}+1)\beta^2 + \sqrt{8\beta^2+1} + 1$ and $n = (10 - 2\sqrt{8\beta^2+1})\beta^3 - 4\beta$, therefore, $(m-n) = (2\beta+1)(\beta-1)^2(4\beta + \sqrt{8\beta^2+1}+1) > 0$

well as economic literature), tells us that it is less costly to produce a good and a service in a bundle rather than independently (saving costs for the packaging, for instance), so we expect that prices follow this cost saving. Here this is the case, but in the second case below, this argument doesn't hold anymore.

In the second area, above to the full line curve (where $\beta > \hat{\beta}_x(\alpha)$), the price of the separate good is lower than as component of the bundle and at the same time, the service price is less costly in the bundle compared to its separate price. Indeed:

$$\begin{aligned} p^B &\gtreqless p^{IP} \text{ if } \beta \lesseqgtr \hat{\beta}_x(\alpha) \\ r^B &< r^{IP}, \forall \alpha \in [0,1] \text{ and } \beta \in [0, \frac{1}{2}] \end{aligned}$$

and here $\hat{\beta}_x(\alpha) = \frac{1}{4\alpha}(\sqrt{8\alpha^2 + 1} - 1)$.

It is interesting to see how, with increasing costs, the price of the independent good can be lower than the tied good. The introduction of increasing costs has a strong impact on the good price since results of section 5 are reversed. Nevertheless, there is no impact on the service price which is always lower in the bundle than sold separately.

When the good and the related service are perfectly complementary ($\beta = \frac{1}{2}$), we are in the second area. The intuition suggests that the bundling strategy is more profitable when the components of the bundle are complementary. It is straightforward for the monopoly to charge a high price for the good in the bundle. As the bundling strategy allows monopoly to practice price discrimination, consumers who have an intensive use of the good accept this high price. Consumers have more utility by consuming the good in relation to the service. The monopoly can charge a low price for the service as component of the bundle compared to the separates sales. Even if the price of the bundle is higher than the sum of the separates sales, the bundling strategy can be an effective strategy. Some consumers consider that it is better to spend a large additional sum instead of making themselves the bundle. When the service is purely optional, $\alpha = 0$ ($\forall \beta < \frac{1}{2}$), results are the same.[29]

6.4 Comparative statics

Bundling strategy allows producers to restrict their unit production costs, with costs saving on packaging or storage for instance, or more generally, on the "selling costs". Simultaneously, when products are perfect complements, bundling literature advocates a high price for the good and the service sold in the bundle. In our analysis, the optionality of the service plays an important role and has an impact on the marginal price of the independent good. In this case, the good sold in the bundle is higher than sold independently.

With complementary goods, the analysis of our model is not as explicit as those of Martimort (1992) where utility function is symmetric. Martimort shows that the independent pricing strategy is inefficient for consumers, since it is costly to sign different contracts to different shops. Here, it is not explicitly the case as the price of the independent good can be lower than the price of the tied good. The optionality of the service as well as the increasing costs emphasize that independent pricing strategy can be effective from consumers' point of view. The overall analysis is given by the figure 2.

In this graphic $\bar{\beta}_s(\alpha) = \{\beta \in [0, \frac{1}{2}] \mid r^{IP} = r^F\}$.

[29] See appendix D.2.

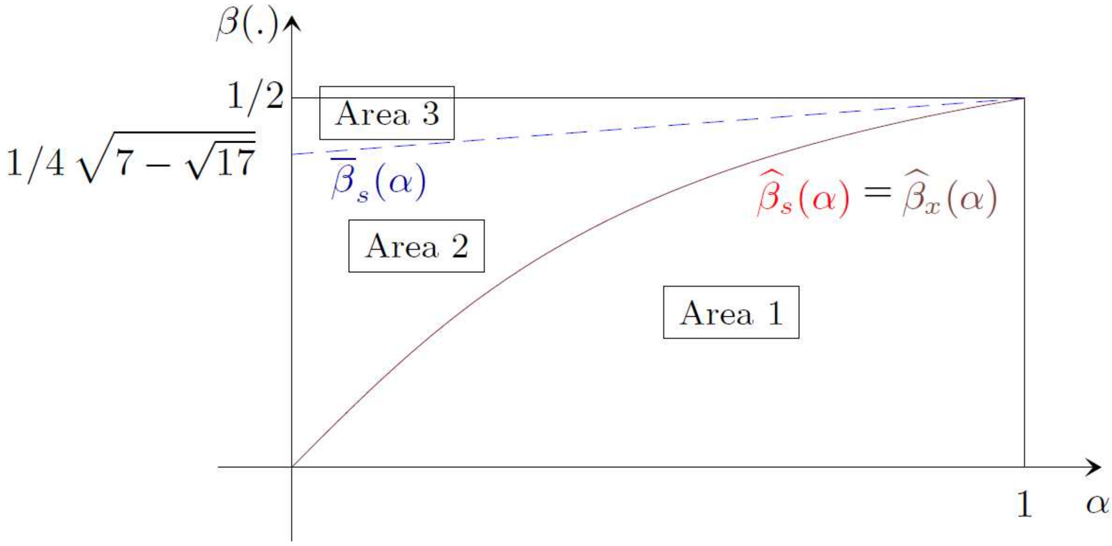

Figure 2: Comparison between prices with independent pricing, bundling strategies and the first-best situation

The following table shows possible outcomes according to the different areas in relation to the type of strategies:

- FB: Complete Information situation ("First-Best situation")
- IP: Independent Pricing strategy
- VL: Bundling strategy

	IP/FB	VL/FB	IP/VL
Area 1	$p^{IP}(\theta) > p^F(\theta)$ $r^{IP}(\theta) > r^F(\theta)$	$p^B(\theta) > p^F(\theta)$ $r^B(\theta) > r^F(\theta)$	$p^{IP}(\theta) > p^B(\theta)$ $r^{IP}(\theta) > r^B(\theta)$
Area 2	$p^{IP}(\theta) > p^F(\theta)$ $r^{IP}(\theta) > r^F(\theta)$	$p^B(\theta) > p^F(\theta)$ $\mathbf{r}^B(\boldsymbol{\theta}) < \mathbf{r}^F(\boldsymbol{\theta})$	$\mathbf{p}^{IP}(\boldsymbol{\theta}) < \mathbf{p}^B(\boldsymbol{\theta})$ $r^{IP}(\theta) > r^B(\theta)$
Area 3	$p^{IP}(\theta) > p^F(\theta)$ $\mathbf{r}^{IP}(\boldsymbol{\theta}) < \mathbf{r}^F(\boldsymbol{\theta})$	$p^B(\theta) > p^F(\theta)$ $\mathbf{r}^B(\boldsymbol{\theta}) < \mathbf{r}^F(\boldsymbol{\theta})$	$\mathbf{p}^{IP}(\boldsymbol{\theta}) < \mathbf{p}^B(\boldsymbol{\theta})$ $r^{IP}(\theta) > r^B(\theta)$

Table 2.1: possible outcomes in relation to different areas

In order to understand the figure 2 and table 2.1., we comment each areas.

In area 1, the standard case occurs: the information cost is taken into account for the monopoly, so that marginal prices are ranked with respect to the information cost.

In area 2, the information cost falls significantly the marginal price of the service and it becomes smaller than its marginal cost. This effect can be viewed as an "informational dumping effect".[30] There is a trade-off between the degree of complementarity and the

[30]Dumping is often defined as the ability of a monopoly to fix a lower price in order to force the potential entrants to revise downwards their estimations of profitability on this market. Ex-ante, this allows the monopoly to deter potential competitors to enter the market.

degree of the optional service. The monopoly lowers its bundle price ("price effect") but it makes up for quantities sold: the quantity effect dominates the price effect. However, the dumping effect appears even when the degree of service is not totally optional to satisfy the essential need. Moreover as evoked in the previous section, bundling strategy allows monopoly to practice price discrimination, therefore consumers who have an intensive use of the good accept a higher price. This dumping effect would possible because of using two-part tariff, the monopoly can consequently rise the fixed part.

Analytically, the area 3 (where $\beta > \bar{\beta}_s(\alpha)$) is the most interesting one since the dumping effect appears for the service in both independent pricing and bundling strategies. As the service is optional, its price is lower than its marginal cost. This dumping effect was underlined in the bundling case, nevertheless when the service is purely optional, the degree of complementarity must be high enough to enjoy this discount.[31] Moreover, this analysis holds when the components are perfect complements ($\beta = \frac{1}{2}$). The marginal price of service is lower than its marginal cost, moreover the quantities are sub-optimal[32] in relation to the complete information case. As the variable part of the tariff is minimized, the monopoly can compensate this loss through the fixed-fee of the tariff: this effect can be described as a "catching effect".

Moreover, the marginal price of independent good is lower than its price in the bundle. The introduction of increasing costs has an impact on the good marginal price as this case not occurs with constant costs. This price discount for the independent good appears when the good and the service have a high degree of complementarity. With complement goods, the bundle is attractive for many consumers and the monopoly fixes a high marginal price for the good relatively to the independent pricing strategy.

One can not directly conclude that the overall analysis given by Martimort (equation (7) to section 5) holds because asymmetric properties (H1)-(H3) are considered. In our analysis framework, one can conclude that:

$$\begin{aligned} x^F(\theta) &> x^B(\theta) \gtrless x^{IP}(\theta) \\ s^F(\theta) &> s^B(\theta) > s^{IP}(\theta) \end{aligned}$$

and prices can be rewritten as follow:

$$\begin{aligned} p^F(\theta) &< p^{IP}(\theta) \gtrless p^B(\theta) \\ r^F(\theta) &\gtrless r^{IP}(\theta) > r^B(\theta) \end{aligned}$$

As we compare bundling and independent pricing, considering marginal prices only, one can conclude that it is not always less costly for consumers to purchase *each unit* of good and service at two different shops rather than together in a bundle. To have a comparison of the entire cost, we should compare the consumer's rent in each configuration (that is including the fix part of the tariff).

7 Conclusion

This paper analyzes optimal nonlinear pricing when a firm offers a commodity and an attached service. We compare the bundling strategy and the independent pricing one. The profitability of bundling strategy depends on the degree of the optional service and

[31] $\beta = \frac{\sqrt{7-\sqrt{17}}}{4} \approx 0.424$.

[32] See appendix C.3.

the degree of complementary between the good and the associated service. This paper deals with two cases concerning the cost function.

In a first time, we suppose that the shops sell their products at the market prices, they have constant marginal costs. In the general case, one can conclude that the separate marginal prices of the good and the service are always higher than as a part of bundle. As Martimort (1992), one find that it is costly for consumers to buy the good and the service separately due to the implicit competition between the two shops. In the general case, bundling is an optimal strategy for both principals and consumers.

With the specified utility function, we can conclude that with bundling strategy, the optimal quantities are higher than those with an independent pricing strategy. If the service is purely optional $i.e$ when $\alpha = 0$, marginal prices of the good sold independently and in a bundle are the same. However if $\alpha > 0$, this is the standard case where the good in the bundle is lower than separately. For the attached service, whatever its degree of optionality, the marginal price in bundle is always lower than sold independently. Comparing marginal prices, consumers prefer to buy the good and the attached service in a bundle as long as the degree of optionality is positive otherwise consumers are indifferent.

In a second time, we use increasing costs to describe the situation where both shops produce the good and the related service. We show that there is a trade-off between the degree of complementarity and the optionality of the service. The overall analysis (see Figure 2) shows that there are three cases through different areas. Limiting the analysis to the polar cases ($\beta = \{0, \frac{1}{2}\}, \forall \alpha$), the results are the following. With the specified utility function, when the good and the service are independent ($\beta = 0$), if the monopoly commits to a bundling strategy then marginal prices are both lower then the separates sales. The monopoly prefers to leave a significant unit margin and compensates to the quantities sold. Comparing marginal prices, consumers prefer to buy the good and the attached service in a bundle.

However, when the degree of the complementarity becomes sufficiently high (when $\beta > \{\hat{\beta}_x(\alpha), \bar{\beta}_s(\alpha)\}$), the marginal price of the separate good is lower than the good price in the bundle. The intuition suggests that the profitability of a bundling strategy depends on the degree of complementarity between the good and the attached service. It is straightforward for the monopoly to charge a higher price for good in bundle.

In the polar case when good and service are perfectly complementary, contrary to Martimort (1992) we can not conclude that it is costly for consumers to sign two contracts from different shops than to buy the bundle. With the asymmetric function, the marginal price of good can be lower independently than in the bundle whereas the service is more attractive when there is a cooperation between principals. Because of asymmetric properties in the utility function, profitability results of bundling strategy depends, on one hand, on the degree of complementarity between commodity and related service and on the other hand, on the degree of the optional service.

Appendix

A First Best

Without any further calculs, first-best levels for prices and quantities are given. More details could be found in Podesta and Poudou (2008).

A.1 Constant costs

Explicitely, with (H4), quantities of the good and the attached service are given by:

$$x^F(\theta) = \frac{\theta(1+2\beta\alpha) - w(1+2\beta)}{(1-4\beta^2)} \text{ and } s^F(\theta) = \frac{\theta(\alpha+2\beta) - w(1+2\beta)}{(1-4\beta^2)}$$

For a positive level of attached service, we assume a condition on w, i.e. $w < w^0$ where $w^0 = \frac{\theta(\alpha+2\beta)}{1+2\beta}$. The derivative of the utility in relation to x and s gives the energy and relative service prices:

$$\begin{aligned} u_x(x(\theta), s(\theta), \theta) &= p^F(\theta) \Rightarrow p^F(\theta) = w \\ u_s(x(\theta), s(\theta), \theta) &= r^F(\theta) \Rightarrow r^F(\theta) = w \end{aligned}$$

The equilibrium prices in the first-best situation are equal to marginal cost.

A.2 Increasing costs

Under increasing costs assumption (H5), quantities of the good and the attached service are given by $x^F(\theta) = \frac{1+\alpha\beta}{2(1-\beta^2)}\theta$ and $s^F(\theta) = \frac{\alpha+\beta}{2(1-\beta^2)}\theta$. Energy and service marginal prices are given by $p^F(\theta) = x^F(\theta)$ and $r^F(\theta) = s^F(\theta)$.

B Nonlinear tariff and bundling

With bubdling, the firm's objective (EP) can be implemented by an integration by parts:

$$\begin{aligned} \max E(\pi) = &\int_{\underline{\theta}}^{\overline{\theta}} \left[u(x(\theta), s(\theta), \theta) - \varphi(\theta) u_\theta(x(\theta), s(\theta), \theta) - C(x(\theta), s(\theta))\right] f(\theta) d\theta \\ &\text{s.t. } u_{\theta x}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + u_{\theta s}(x(\theta), s(\theta), \theta)\dot{s}(\theta) \geq 0 \end{aligned}$$

which can allow us to write that $\dot{U}(\theta) = u(x(\theta), s(\theta), \theta)$, and at the equilibrium the couple $\{x^*(\theta), s^*(\theta)\}$ satisfies the system:

$$\begin{cases} u_x(x, s, \theta) - \varphi(\theta) u_{\theta x}(x, s, \theta) = c'(x) \\ u_s(x, s, \theta) - \varphi(\theta) u_{\theta s}(x, s, \theta) = c'(s) \end{cases} \tag{9}$$

Since $\varphi(\theta)u_{\theta x} \geq 0$ and $\varphi(\theta)u_{\theta s} \geq 0$, one can conclude that the quantities are sub-optimal compared to the complete information situation (given above) for which $\varphi(\theta)$ would equated to zero for all θ.

B.1 Constant costs

- The first and the second order incentive compatibility constraints implemented with $T(\theta) = T(x(\theta), s(\theta))$ are given by:

$$\begin{aligned} u_x(x, s, \theta)\ \dot{x}(\theta) + u_s(x, s, \theta)\ \dot{s}(\theta) - \dot{T}(\theta) &= 0 \\ u_{\theta x}(x, s, \theta)\ \dot{x}(\theta) + u_{\theta s}(x, s, \theta)\ \dot{s}(\theta) &\geq 0 \end{aligned}$$

- The variation of x^B and s^B in relation to θ with specified utility function is given by:

$$\dot{x}^B_\theta = \frac{1 + 2\beta\alpha}{(1 - 4\beta^2)} \geq 0 \ \text{ and } \dot{s}^B_\theta = \frac{2\beta + \alpha}{(1 - 4\beta^2)} \geq 0$$

B.2 Increasing costs

- The variation of x^B and s^B in relation to θ with the specified utility function is given by:

$$\dot{x}^B_\theta = \frac{1 + \beta\alpha}{2(1 - \beta^2)} \geq 0 \ \text{ and } \dot{s}^B_\theta = \frac{\beta + \alpha}{2(1 - \beta^2)} \geq 0$$

C Nonlinear tariff with the independent pricing strategy

C.1 Supply of good

The shop 1 maximizes its expected profits under the first and the second order incentive constraints with $t(\theta) = t(x(\theta))$.The expected profits of shop 1 can be rewritten as:

$$\max_{U,x,\hat{\theta}_s} E(\pi) = \int_{\underline{\theta}}^{\overline{\theta}} \left[u(x(\theta), s(\hat{\theta}_s(\theta)), \theta) - c(x(\theta)) - U(\theta) - \tau(\hat{\theta}_s(\theta))\right] f(\theta) d\theta$$

$$\dot{U}(\theta) = u_\theta(x(\theta), s(\hat{\theta}_s(\theta)), \theta) \qquad (\lambda)$$

$$U(\underline{\theta}) = 0$$

$$u_s(x(\theta), s(\hat{\theta}_s(\theta)), \theta)\ \dot{s}(\hat{\theta}_s(\theta)) - \dot{\tau}(\hat{\theta}_s(\theta)) = 0 \qquad (\mu) \tag{IC2}$$

where $f(\theta)$ is the density function of consumers preference. At the equilibrium, we have $\hat{\theta}_s(\theta) = \theta$.

From the analysis of Martimort (1992), to solve the program, we write the Hamiltonian under (λ) and (μ) constraints and :

$$\begin{aligned} H(U, x, \hat{\theta}_s) \ = \ & f(\theta)[-c(x) - U - \tau(\hat{\theta}_s) + u(x, s(\hat{\theta}_s), \theta)] \\ & + \lambda(\theta) u_\theta(x, s(\hat{\theta}_s), \theta) \\ & + \mu(\theta)[u_{\theta s}(x, s(\hat{\theta}_s), \theta) \dot{s}(\hat{\theta}_s) - \tau(\hat{\theta}_s)] \end{aligned}$$

The dynamic system of Hamilton-Jacobi (SHJ) in relation to the state variable has a form:

$$\frac{\partial H}{\partial U} = \dot{\lambda}(\theta) \Leftrightarrow \dot{\lambda}(\theta) = f(\theta) \tag{1}$$

Therefore, according to the edge condition $\dot{\lambda}(\theta) = f(\theta)$ and according to the transversal condition we can restate:

$$U(\underline{\theta}) = 0 \text{ and } \lambda(\bar{\theta}) = 0 \tag{10}$$

After have rewritten the edge and transversal conditions we restate the SHJ with respect to the control variables:

$$\begin{aligned}\frac{\partial H}{\partial x} &= f(\theta)[-c'(x(\theta)) + u_x(x(\theta), s(\hat{\theta}_s(\theta)), \theta)] + \lambda(\theta)\left[u_{\theta x}(x(\theta), s(\hat{\theta}_s(\theta)), \theta)\right] \\ &+\mu(\theta)\left[u_{xs}(x(\theta), s(\hat{\theta}_s(\theta)), \theta)\ \dot{s}(\hat{\theta}_s(\theta))\right] = 0\end{aligned} \tag{11}$$

If we suppose that the principals' contracts supply are truthfully ex-post we assume $\hat{\theta}_s = \theta$, we can restate:

$$\begin{aligned}\frac{\partial H}{\partial \hat{\theta}_s}_{|\hat{\theta}_s=\theta} &= f(\theta)[(-\dot{\tau}(\theta) + u_s(x(\theta), s(\theta), \theta)\dot{s}(\theta)) \\ &+\lambda(\theta)(u_{\theta s}(x(\theta), s(\theta), \theta)\dot{s}(\theta)) \\ &+\mu(\theta)(u_{ss}(x(\theta), s(\theta), \theta)(\dot{s}(\theta))^2 + u_s(x(\theta), s(\theta), \theta)\ddot{s}(\theta) - \ddot{\tau}(\theta))] = 0\end{aligned} \tag{12}$$

As the principals offer truthtelling tariff for agents to reveal their true type, the preference for the service is restated $\hat{\theta}_s = \theta$, the derivative second order condition (IC2) with respect to θ is given by:

$$\begin{aligned}&\ddot{s}(\theta)u_s(x(\theta), s(\theta), \theta) + u_{ss}(x(\theta), s(\theta))(\dot{s}(\theta))^2 - \ddot{\tau}(\theta) \\ &+u_{\theta s}(x(\theta), s(\theta), \theta)\dot{s}(\theta) + u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta)\dot{s}(\theta) = 0\end{aligned} \tag{13}$$

If the solution is separating then $\dot{x}(\theta) \neq 0, \dot{s}(\theta) \neq 0$, the second order incentive compatibility conditions are satisfied and the relation (1) is given by $\lambda(\theta) = F(\theta) + k$ where k is a constant. As $F(\bar{\theta}) = 1$, thus:

$$\begin{aligned}\lambda(\bar{\theta}) &= F(\bar{\theta}) + k = 0 \Rightarrow k^* = -1 \\ \lambda^*(\theta) &= F(\theta) - 1 = -(1 - F(\theta))\end{aligned} \tag{14}$$

From the equations (13) and (14) into (12), we can restate the SHJ constraints:

$$-(1 - F(\theta))u_{\theta s}(x(\theta), s(\theta), \theta)\dot{s}(\theta) + \mu(\theta)\left[-u_{\theta s}(x(\theta), s(\theta), \theta)\dot{s}(\theta) - u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta)\dot{s}(\theta)\right] = 0$$

$$\mu(\theta) = -(1 - F(\theta))\frac{u_{\theta s}(x(\theta), s(\theta), \theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + u_{\theta s}(x(\theta), s(\theta), \theta)} \tag{15}$$

Under the analysis of Martimort (1992), by replacing the equation (15) into the equation (11), the SHJ can be restated as following:

$$\begin{aligned}&f(\theta)[(u_x(x(\theta), s(\theta), \theta) - c'(x(\theta))) + (-(1 - F(\theta))u_{\theta x}(x(\theta), s(\theta), \theta)) \\ &+(-(1 - F(\theta))\frac{u_{\theta s}(x(\theta), s(\theta), \theta)u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + u_{\theta s}(x(\theta), s(\theta), \theta)})] = 0\end{aligned} \tag{16}$$

It is possible to assume $\varphi(\theta) = \frac{1-F(\theta)}{f(\theta)}$ the equation (16) can be rewritten as: With some simplifications:

$$\begin{aligned}&-c'(x(\theta)) + u_x(x(\theta), s(\theta), \theta) - \varphi(\theta)u_{\theta x}(x(\theta), s(\theta), \theta)- \\ &-\varphi(\theta)\frac{u_{\theta s}(x(\theta), s(\theta), \theta)u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + u_{\theta s}(x(\theta), s(\theta), \theta)} = 0\end{aligned} \tag{17}$$

with $x(\overline{\theta}) = x^F(\overline{\theta})$. With the specified utility (H6), the SHJ can be rewritten as -omitting arguments-:

$$2\theta - \bar{\theta} - 2x + 2\beta s - (\bar{\theta} - \theta)\frac{2\alpha\beta\ \dot{s}}{2\beta\ \dot{x} + \alpha} = 0 \tag{18}$$

C.2 Supply of service

The shop 2 maximizes its expected profits with $\tau(\theta) = \tau(s(\theta))$. Here, the tariff depends on the quantity of service chosen, thus shop 2 maximizes its expected profits under the first and the second order incentive constraints:

$$\max_{U,s,\hat{\theta}_x} E(\pi) = \int_{\underline{\theta}}^{\overline{\theta}} \left[u(x(\hat{\theta}_x(\theta)), s(\theta), \theta) - c(s(\theta)) - U(\theta) - t(\hat{\theta}_x(\theta))\right] f(\theta)d\theta$$

$$\dot{U}(\theta) = u(x(\hat{\theta}_x(\theta)), s(\theta), \theta) \qquad (\lambda)$$

$$U(\underline{\theta}) = 0$$

$$u_x(x(\hat{\theta}_x(\theta)), s(\theta), \theta)\dot{x}(\hat{\theta}_x(\theta)) - \dot{t}(\hat{\theta}_x(\theta)) = 0 \qquad (\mu) \tag{IC2}$$

where $f(\theta)$ is the density function of consumers preferences. At the equilibrium, we have $\hat{\theta}_x(\theta) = \theta$. To solve the program, we write the Hamiltonian under (λ) and (μ) constraints and under the analysis of Martimort (1992):

$$\begin{aligned} H(U, s, \hat{\theta}_x) \quad = \quad & f(\theta)[-c(s) - U - t(\hat{\theta}_x) + u(x(\hat{\theta}_x), s, \theta)] \\ & +\lambda(\theta)u_\theta(x(\hat{\theta}_x), s, \theta) \\ & +\mu(\theta)[u_x(x(\hat{\theta}_x), s, \theta)\dot{x}(\hat{\theta}_x) - t(\hat{\theta}_x)] \end{aligned}$$

Here relations (1), (10) still holds and the optimal path of service is such that:

$$\begin{aligned} \frac{\partial H}{\partial s} = f(\theta)[-c'(s(\theta)) + u_s(x(\hat{\theta}_x(\theta)), s(\theta), \theta)] + \lambda(\theta)\left[u_{\theta s}(x(\hat{\theta}_x(\theta)), s(\theta), \theta)\right] \\ +\mu(\theta)\left[u_{xs}(x(\hat{\theta}_x(\theta)), s(\theta), \theta)\dot{x}(\hat{\theta}_x(\theta))\right] = 0 \end{aligned} \tag{19}$$

If we suppose that the principals' contracts supply are truthfully ex-post we assume $\hat{\theta}_x = \theta$, we can restate:

$$\begin{aligned} \frac{\partial H}{\partial\hat{\theta}_x}_{|\hat{\theta}_x=\theta} = f(\theta)[(-\dot{t}(\theta) + u_x(x(\theta), s(\theta), \theta)\dot{x}(\theta)) \\ +\lambda(\theta)(u_{\theta x}(x(\theta), s(\theta), \theta)\dot{x}(\theta)) \\ +\mu(\theta)(u_{xx}(x(\theta), s(\theta), \theta)(\dot{x}(\theta))^2 + u_x(x(\theta), s(\theta), \theta)\ddot{x}(\theta) - \ddot{t}(\theta))] = 0 \end{aligned} \tag{20}$$

As the principals offer truthtelling tariff for agents to reveal their true type, the preference for the service is restated $\hat{\theta}_x = \theta$, the derivative second order condition (IC2) with respect to θ is given by:

$$\begin{aligned} \ddot{x}(\theta)u_x(x(\theta), s(\theta), \theta) + u_{xx}(x(\theta), s(\theta), \theta)(\dot{x}(\theta))^2 - \ddot{t}(\theta) \\ +u_{\theta x}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta)\dot{s}(\theta) = 0 \end{aligned} \tag{21}$$

If the solution is separating then $\dot{s}(\theta) \neq 0, \dot{x}(\theta) \neq 0$, the second order incentive compatibility conditions are satisfied and (14) holds again. From equations (14) and (21) into (20), we can restate the SHJ constraints:

$$-(1-F(\theta))u_{\theta x}(x(\theta), s(\theta), \theta)\dot{x}(\theta) + \mu(\theta)\left[-u_{\theta x}(x(\theta), s(\theta), \theta)\dot{x}(\theta) - u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta)\dot{s}(\theta)\right] = 0$$

$$\mu(\theta) = -(1-F(\theta))\frac{u_{\theta x}(x(\theta), s(\theta), \theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta) + u_{\theta x}(x(\theta), s(\theta), \theta)} \tag{22}$$

By replacing equation (22) into (19), the SHJ can be restated as following:

$$\begin{aligned} f(\theta)[(u_s(x(\theta), s(\theta), \theta) - c'(s(\theta))) + (-(1-F(\theta))u_{\theta s}(x(\theta), s(\theta), \theta)) \\ +(-(1-F(\theta))\frac{u_{\theta x}(x(\theta), s(\theta), \theta)(u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta) + u_{\theta x}(x(\theta), s(\theta), \theta)})] = 0 \end{aligned} \tag{23}$$

and equation (23) rewrites:

$$\begin{aligned} -c'(s(\theta)) + u_s(x(\theta), s(\theta), \theta) - \varphi(\theta)u_{\theta s}(x(\theta), s(\theta), \theta) - \\ -\varphi(\theta)\frac{u_{\theta x}(x(\theta), s(\theta), \theta)u_{xs}(x(\theta), s(\theta), \theta)\dot{x}(\theta)}{u_{xs}(x(\theta), s(\theta), \theta)\dot{s}(\theta) + u_{\theta x}(x(\theta), s(\theta), \theta))} = 0 \end{aligned} \tag{24}$$

with $s(\bar{\theta}) = s^F(\bar{\theta})$. Explicitely (24) writes:

$$\alpha(2\theta - \bar{\theta}) - 2s + 2\beta x - (\bar{\theta} - \theta)\frac{2\beta\ \dot{x}}{2\beta\ \dot{s} + 1} = 0$$

C.3 The problem solving system

- We can rewritte the equation (4) as the differential equations system following:

$$\begin{cases} \dot{x} = \frac{u_{\theta s}(x,s,\theta)}{u_{xs}(x,s,\theta)}\dfrac{[u_s(x,s,\theta) - \varphi(\theta)u_{\theta s}(x,s,\theta) - c'(s)]\,[u_x(x,s,\theta) - c'(x)]}{\Gamma(x,s,\theta)} \\ \dot{s} = \frac{u_{\theta x}(x,s,\theta)}{u_{xs}(x,s,\theta)}\dfrac{[u_x(x,s,\theta) - \varphi(\theta)u_{\theta x}(x,s,\theta) - c'(x)]\,[u_s(x,s,\theta) - c'(s)]}{\Gamma(x,s,\theta)} \end{cases} \tag{25}$$

  where

$$\begin{aligned} \Gamma(x,s,\theta) &= [u_s(x,s,\theta) - c'(s)]\,[u_x(x,s,\theta) - c'(x) - \varphi(\theta)u_{\theta x}(x,s,\theta)] \\ &\quad - [u_x(x,s,\theta) - c'(x)]\,\varphi(\theta)u_{\theta s}(x,s,\theta) \end{aligned}$$

  with $\Gamma(x^F(\bar{\theta}), s^F(\bar{\theta}), \bar{\theta}) = 0$.

We do not attempt to investigate the global analysis of (25), but we first try to define the solution when the service is purely optional that is if ($u_{\theta s} \equiv 0$). In this situation, (25) becomes:

$$\begin{cases} -c'(x) + u_x(x,s,\theta) - \varphi(\theta)u_{\theta x}(x,s,\theta) = 0 \\ -c'(s) + u_s(x,s,\theta) - \varphi(\theta)\dfrac{u_{\theta x}(x,s,\theta)\ u_{xs}(x,s,\theta)\ \dot{x}}{u_{xs}(x,s,\theta)\ \dot{s} + u_{\theta x}(x,s,\theta)} \end{cases}$$

we see from the first equation in (4) that $x = x^B(s)$ so we know the "reaction" function $x^B(s)$ is increasing[33] that is $x^{B\prime}(s) > 0$ so (4) writes knows:

$$\begin{cases} x = x^B(s) \\ -c'(s) + u_s(x,s,\theta) - \varphi(\theta)\dfrac{u_{\theta x}(x,s,\theta)\ u_{xs}(x,s,\theta)\ x^{b\prime}(s)\dot{s}}{u_{xs}(x,s,\theta)\ \dot{s} + u_{\theta x}(x,s,\theta)} = 0 \end{cases}$$

If second order incentive compatibility conditions are satisfied, such that $\dot{x}(\theta), \dot{s}(\theta) > 0$, it must be true that $u_s(x,s,\theta) = c'(s) + \varphi(\theta)\frac{u_{\theta x}(x,s,\theta)\ u_{xs}(x,s,\theta)\ x^{b\prime}(s)\dot{s}}{u_{xs}(x,s,\theta)\ \dot{s}+u_{\theta x}(x,s,\theta)} > c'(s)$ hence[34] $s^{IP}(\theta) \leq s^B(\theta)$ and $x^{IP}(\theta) = x^b(s^{IP}(\theta)) \leq s^B(\theta) = x^b(s^B(\theta))$ since $x^{b\prime}(s) > 0$).

- The quantities of good and service are given by the equation (5) in section 4.3. At the equilibrium, the couple $\{\eta_x^*, \eta_s^*\}$ is given by:

$$\begin{aligned} \eta_x^* &= -\frac{\alpha\sqrt{1+32\,\beta^2}(1+4\,\beta^2) + 4\beta^2\alpha(\sqrt{1+32\,\beta^2}+7) + 4\,\beta(8\beta^2+1) - \alpha}{\left(1-4\,\beta^2\right)X} \\ \eta_s^* &= -\frac{\sqrt{1+32\,\beta^2}(1+2\,\beta\,\alpha) + 2\,\beta\,(4\,\beta+\alpha) - 1}{4\beta\ \left(1-4\,\beta^2\right)} \end{aligned} \tag{26}$$

where $X = 2\beta\left(3 + \sqrt{1+32\,\beta^2}\right)$.

C.4 Increasing costs

- At the equilibrium, the couple $\{\kappa_x^*, \kappa_s^*\}$ is given by:

$$\begin{aligned} \kappa_x^* &= -\frac{2(\sqrt{8\,\beta^2+1}-1)\beta + (\sqrt{8\,\beta^2+1}-7)\alpha\beta^2 + (1+\sqrt{8\,\beta^2+1})\alpha - 4\beta^3}{D} \\ \kappa_s^* &= -\frac{(\sqrt{8\,\beta^2+1}-1)\beta\alpha - 2\beta^2 + \sqrt{8\,\beta^2+1}+1}{4\left(1-\beta^2\right)\beta} \end{aligned}$$

where $D = 2\left(1-\beta^2\right)(3-\sqrt{8\,\beta^2+1})\beta$. For the values of parameters, κ_x and κ_s are negative.

- The comparison between marginal prices with an independent pricing strategy and in the fist-best situation, under a uniform law and with the specific utility funtion, is given by:

$$\begin{aligned} p^{IP}(\theta) &= p^F(\theta) + \frac{1}{X}(\bar{\theta}-\theta)\left[\begin{array}{c} -8\,\beta^4\alpha - (10+2\sqrt{8\,\beta^2+1})\beta^3 \\ -(\sqrt{8\,\beta^2+1}-1)3\beta^2\alpha + 4\beta - (1-\sqrt{8\,\beta^2+1})\alpha \end{array}\right] \\ r^{IP}(\theta) &= r^F(\theta) + \frac{1}{X}(\bar{\theta}-\theta)\left[\begin{array}{c} -8\,\beta^4 - (10+2\sqrt{8\,\beta^2+1})\beta^3\alpha \\ -(\sqrt{8\,\beta^2+1}-1)3\beta^2 + 4\beta\alpha - 1 + \sqrt{8\,\beta^2+1} \end{array}\right] \end{aligned}$$

[33] Where $x^{b\prime}(s) = -\frac{u_{xs}(x,s,\theta)-\varphi(\theta)u_{\theta xs}(x,s,\theta)}{u_{xx}(x,s,\theta)-\varphi(\theta)u_{\theta xx}(x,s,\theta)-c''(x)}$, indeed $x^b(\theta) = x^b(s^B(\theta))$. Notice that if the service is purely optional ($u_{\theta s} = u_{\theta xs} \equiv 0$) then $x^{b\prime}(s) > 0$ since $u_{\theta xx}(x,s,\theta) > 0$ and cost are convex.

[34] We have not prove that this allocation is not unique (as suspected from the analysis of Martimort, 1992) and incentive compatible. However, we admit this is the case.

D Comparison between IP and bundling strategies

D.1 Constant costs

- The quantities comparison between the situation where the monopoly produces a bundle and when he/she buys its goods independently is given by:

$$\begin{aligned} x^{IP}(\theta) &= x^B(\theta) - \frac{\bar{\theta}-\theta}{(1-4\beta^2)X}\left[(\alpha+2\beta)\left(\sqrt{1+32\,\beta^2}-1\right)+16\alpha\beta^2\right] \\ s^{IP}(\theta) &= s^B(\theta) - \frac{\bar{\theta}-\theta}{(1-4\beta^2)X}\,(1+2\beta\alpha)\left(\sqrt{1+32\,\beta^2}-1\right) \end{aligned}$$

  One can directly conclude that $x^{IP}(\theta) < x^B(\theta) < x^F(\theta)$ and $s^{IP}(\theta) < s^B(\theta) < s^F(\theta)$. With a bundling strategy, optimal quantities of good and service are higher than with an independent pricing strategy. However, these both quantities are sub-optimal in relation to the first-best situation.

D.2 Increasing costs

- When the service is purely optional, $\alpha = 0$ thus $\forall\beta < \frac{1}{2}$, the marginal price of good is always smaller in bundle than separatly. With a uniform law on $[1,2]$ we can rewrite $p^{IP} = p^B + \frac{1}{D}(-4\beta^2 - \sqrt{1+8\,\beta^2}+1)$. The marginal price of service depends on its optional character in order to satisfy the primary need but also to the degree of complementarity between the commodity and the contingent service. However when the service is purely optional, its marginal price is always higher than under the package form. With a uniform law on $[1,2]$, we can rewrite as follows $r^{IP} = r^B + \frac{1}{D}\left[-(2\beta^2-1)(4\beta^2+\sqrt{1+8\,\beta^2}-1)\right]$.

References

[1] Adams W. J. and Yellen J. L. (1976). "Commodity Bundling and the Burden of Monopoly", *Quarterly Journal of Economics*, vol. 91, pp. 475-498.

[2] Bateson J. E. (1995). *Managing Services Marketing,* The Dryden Press.

[3] Burstein M. L. (1960). "The Economics of Tie-In Sales", *The Review of Economics and Statistics*, vol. 42, pp. 68-73.

[4] Martimort D. (1992). "Multi-principaux avec Anti-selection", *Annales d'économie et de statistique,* vol. 28.

[5] Martimort D. (1996). "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory", *RAND Journal of Economics*, vol. 27 , pp. 1-31.

[6] Maskin E. and Riley J. (1984). "Monopoly with Incomplete Information", *RAND Journal of Economics*, vol. 15, pp. 171-196.

[7] McAfee R. P., McMillan J. and Whinston M. D. (1989). "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values", *Quarterly Journal of Economics*, vol. 104, pp. 371-383.

[8] Mussa M. and Rosen S. (1978). "Monopoly and Product Quality", *Journal of Economic Theory*, vol. 18, pp. 301-317.

[9] Nalebuff B. (2004). "Bundling as an Entry Barrier", *Quarterly Journal of Economics*, vol. 119, pp. 159-187.

[10] Olsen T. and Osmundsen P. (2001). "Strategic Tax Competition; Implications of National Ownership", *Journal of Public Economics*, vol. 81, pp. 253-277.

[11] Podesta M. and Poudou J.-C. (2008) "Optimal nonlinear pricing, bundling commodities and contingent services", *Working Paper CREDEN*, 08.04.76 (www.creden.univ-montp1.fr/fr/publications.html)

[12] Schmalensee R. (1982). "Commodity Bundling by Single-Product Monopolies", *Journal of Law and Economics*, vol. 15, pp. 67-71.

[13] Schmalensee R. (1984). "Gaussian Demand and Commodity Bundling", *Journal of Business*, vol. 57, pp. 211-230.

[14] Wilson R. (1993). *Nonlinear Pricing*, Oxford University Press.